

Câu 1: (3 điểm)

a. Tìm hàm tương quan chéo $\varphi_{12}(\tau)$ của tín hiệu $x_1(t)$, $x_2(t)$.

$$0 \leq \tau < 1: \quad \varphi_{12}(\tau) = \int_{-1+\tau}^0 (t+2)dt + \int_0^{1+\tau} (-t+2)dt = 3 - \tau^2 \quad (0,25đ)$$

$$1 \leq \tau < 3: \quad \varphi_{12}(\tau) = \int_{-1+\tau}^2 (-t+2)dt = \frac{\tau^2}{2} - 3\tau + \frac{9}{2} \quad (0,25đ)$$

$$-1 < \tau \leq 0: \quad \varphi_{12}(\tau) = 3 - \tau^2$$

$$-3 < \tau \leq -1: \quad \varphi_{12}(\tau) = \frac{\tau^2}{2} + 3\tau + \frac{9}{2} \quad (0,25đ)$$

$$\text{Kết luận:} \quad \varphi_{12}(\tau) = \begin{cases} 3 - |\tau|^2 & 0 \leq |\tau| < 1 \\ \frac{|\tau|^2}{2} - 3|\tau| + \frac{9}{2} & 1 \leq |\tau| < 3 \end{cases} \quad (0,25đ)$$

b. Hãy vẽ tín hiệu $x(t)$. (0,5đ)

c. Vẽ $y(t)$ (0,5đ)

tính công suất của tín hiệu $y(t)$.

$$P_y = \frac{1}{6} \cdot 2 \cdot \left| \int_{-2}^{-1} (t+2)^2 dt + \int_{-1}^0 (3+t)^2 dt \right| = 2 \quad (0,5đ)$$

Câu 2: (2 điểm) Tìm, biết $x(t) = \cos(3t)u(t)$, $y(t) = e^{-2t} \cdot u(t)$.

$$X(s) = \frac{s}{s^2 + 9}, Y(s) = \frac{1}{s+2} \quad (0,25đ + 0,25đ)$$

$$Z(s) = \frac{s}{s^2 + 9} \cdot \frac{1}{s+2} = \frac{-2}{13} \cdot \frac{1}{s+2} + \frac{1}{4-6j} \cdot \frac{1}{s+3j} + \frac{1}{4+6j} \cdot \frac{1}{s-3j} \quad (1đ)$$

$$z(t) = \frac{-2}{13} \cdot e^{-2t} u(t) + 0,276 \cdot \cos(3t - 56^\circ) \cdot u(t)$$

$$\text{Hoặc } z(t) = \frac{-2}{13} \cdot e^{-2t} u(t) + 0,276 \cdot \cos(3t - 56^\circ) \cdot u(t) \quad (1đ)$$

Câu 3 : Tìm và vẽ phổ của $x_1(t)$, $x_2(t)$.

$$X_1(\omega) = 4 \prod \left(\frac{\omega}{2\omega_1} \right) \quad (0,5đ) \quad \text{Vẽ } X_1(\omega) \quad (0,25đ)$$

$$X_2(\omega) = \frac{\pi}{2} \sum_{n=-\infty}^{\infty} Sa^2\left(\frac{n\pi}{4}\right) \delta(\omega - n\omega_0) \quad (1đ) \quad \text{.Vẽ } X_2(\omega) \quad (0,25đ)$$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} Sa^2\left(\frac{n\pi}{4}\right) \Pi\left(\frac{\omega - n\omega_0}{\frac{\omega_0}{2}}\right) \quad (0,75đ) \quad \text{.Vẽ } Y(\omega), H(\omega) \quad (0,25đ + 0,25đ)$$

$$Z(\omega) = \frac{B}{2} \Pi\left(\frac{\omega - 2\omega_0}{\frac{\omega_0}{2}}\right) + \frac{B}{2} \Pi\left(\frac{\omega + 2\omega_0}{\frac{\omega_0}{2}}\right) + \frac{B}{2} \Lambda\left(\frac{\omega - 2\omega_0}{\frac{\omega_0}{4}}\right) + \frac{B}{2} \Lambda\left(\frac{\omega + 2\omega_0}{\frac{\omega_0}{4}}\right) \quad (0,25đ) \quad \text{. Vẽ } Z(\omega) \quad (0,25đ)$$

$$Z(\omega) = \frac{2}{\pi^2} \left[\Pi\left(\frac{\omega - 2\omega_0}{\frac{\omega_0}{2}}\right) + \Pi\left(\frac{\omega + 2\omega_0}{\frac{\omega_0}{2}}\right) + \Lambda\left(\frac{\omega - 2\omega_0}{\frac{\omega_0}{4}}\right) + \Lambda\left(\frac{\omega + 2\omega_0}{\frac{\omega_0}{4}}\right) \right] \quad (0,5đ)$$

$$z(t) = \frac{\omega_0}{\pi^3} \cdot \cos(2\omega_0 t) \cdot \left(Sa \frac{\omega_0 t}{4} + \frac{1}{2} Sa^2 \frac{\omega_0 t}{4} \right) \quad (0,75đ)$$